Quantitative model : Ordinary Differential Equations (ODEs)

Hypothesis : the mass action kinetics is acting on all molecular species.

They allow to express the rate of production of a given component as a function of the concentration of other elements of the system.

General form:

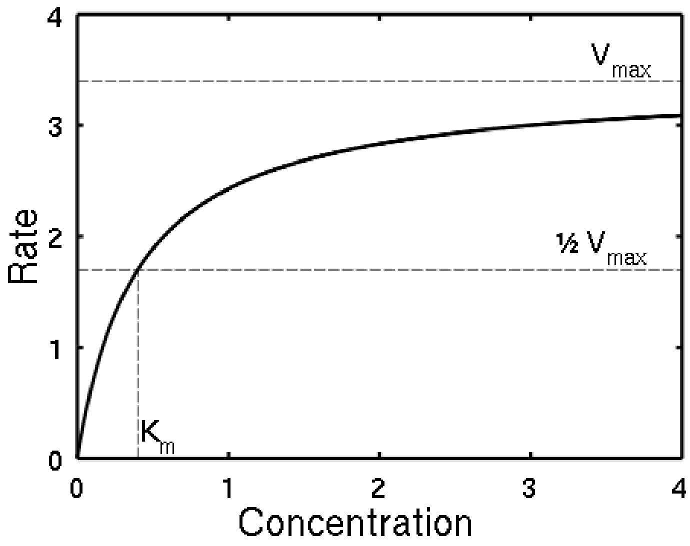
$$\frac{dx}{dt} = \text{synthesis}(x)\text{-degradation}(x)$$

Michaelis-Menten kinetics

The model is an equation describing the rate of enzymatic reactions when the reaction is catalyzed by one enzyme acting on an unique substrate to give a product.

$$\frac{d[P]}{dt} = v_{\max} \frac{[X]}{[X] + K_m}$$

Where P is the product, X the substrate, v_{max} is the maximal synthesis rate of P and K_m is the required concentration of X for half-maximal synthesis rate ($v_{max}/2$)

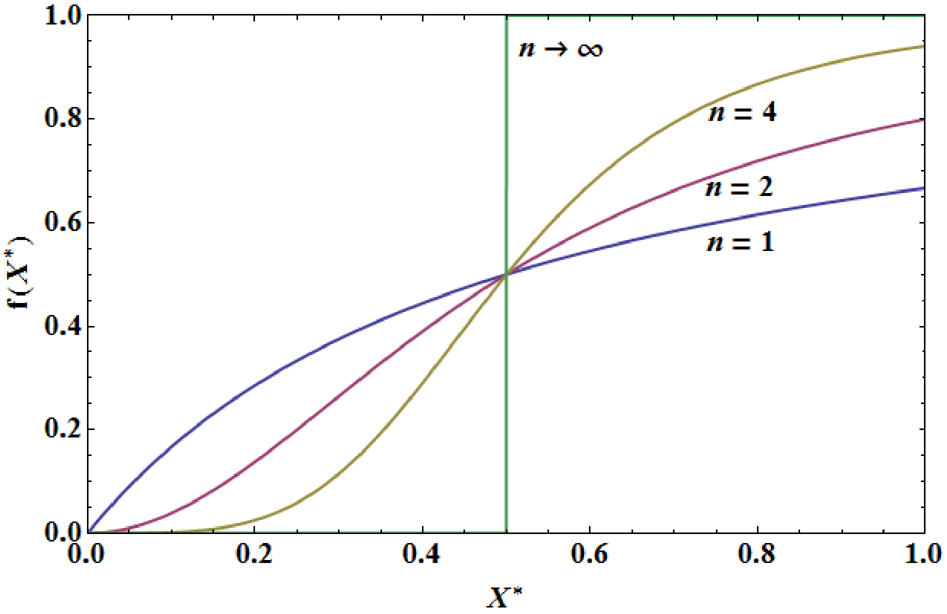


Hill function - Hill kinetics

The Hill function can be derived from statistical mechanics of binding and is often used as an approximation for the input function when the production rate is a function. The input function describes the effect of transcription factor concentration on the production rate of a gene.

$$p = \frac{\beta_{\max} X^n}{K^n + X^n}$$

With β_{max} is maximal transcription rate of the promoter-transcription factor complex, X = activator concentration, K is the activation coefficient and n is the Hill coefficient



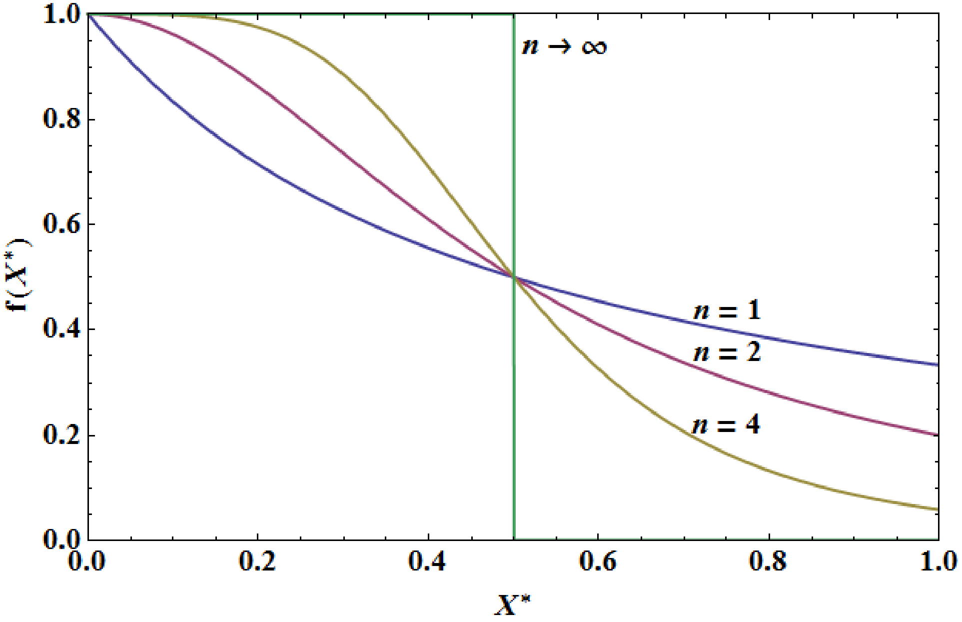
Example with $\beta_{max} = 1$, $K = 0.5$, $X \in [0,1]$

The Hill coefficient comes from the fact the transcription factors can act as multimeres which leads to cooperative behaviour. Typical values for n are 1−4

Hill function - Hill kinetics

For repressors the Hill function decreases with the concentration of active repressor X

$$p = \frac{\beta_{\max}}{1 + \left(\frac{X}{K} \right)^{n}}$$



Example with β_{max} = 1, K = 0.5, $X \in [0,1]$